Structural Approach to Credit Risk

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Corporate Liabilities as Contingent Claims

- The starting point of the models is to take as given the evolution of the market value of a firm's assets and to view all corporate securities as contingent claims on these assets.

Merton's model

- Firm's value follows a geometric Brownian motion: $dV_t = \mu V_t dt + V_t \sigma_t dW_t$ where W is a standard Brownian motion under P.
- Hence, the value of the firm: $V_t = V_0 e =^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t}$
- Under Q, $V_t = V_0 e^{(r - \frac{1}{2}\sigma^2)t + \sigma W_t^Q}$
- Assume that the firm at time 0 has issued two types of claims: debt and equity:
  - Debt is a zero-coupon bond with face value D. Payoff to debt at T: $B_T = \min\{D, V_T\} = D - \max\{D - V_T, 0\}$
  - Equity at time T is $S_T = \max\{V_T - D, 0\}$

Valuation

- How are debt and equity valued prior to maturity date T?
  - Value of the firm $V_T = B_T + S_T$
  - Debt is the difference between riskless bond and put option
  - Equity is a call option on the firm's assets
- Given the current level V, volatility of the asset σ_V and riskless rate r:

- equity: $S_t = C^{BS}(V_t, D, \sigma_V, r, T-t)$
- debt: $B_t = De^{-r(T-t)} - P^{BS}(V_t, D, \sigma_V, r, T-t)$

- The **distance to default** (DD) is computed as the number of standard deviations between the expected asset value at maturity V_T and the liability: $DD = d_2$.

- The **probability of default** (PD) is defined as the probability that the asset value falls below the liability at the end of the time horizon T: $PD = N(-d_2)$

Debt

- Merton's framework handles subordination:

	$V_T < D_S$	$D_S \leq V_T < D_S + D_J$	$D_S + D_J < V_T$
Senior	V_T	D_S	D_S
Junior	0	$V_T - D_S$	D_J
Equity	0	0	$V_T - (D_S + D_J)$

Estimating Asset Value and Asset Volatility

- Asset value and, let alone asset volatility of a firm are rarely observed. They are estimated from equity values assuming the face value of debt is known.

- Since $S_t = C(V_t, D, \sigma_V, r, T-t)$, $dS_t = (...)dt + \frac{\partial C}{\partial V}\sigma_V V_t dW_t$

- Hence, the observed equity volatility $\sigma_S(t) = \frac{\partial C}{\partial V}\sigma_V \frac{V_t}{S_t}$

- Given D, if we observe equity values S_t and volatility $\sigma_S(t)$ we can determine σ_V and V by solving:

$$S_t = C(V_t, D, \sigma_V, r, T-t) \quad (1)$$

$$\sigma_S = \frac{V_t}{S_t}\frac{\partial C}{\partial V}\sigma_V \quad (2)$$

If the leverage significantly changes over the sample period

- Assuming that we observe equity prices $S_{t_0}, S_{t_1}, ..., S_{t_N}$ and let $V_{t_i}(\sigma)$ the value of the assets obtained by inverting Black-scholes at time t_i for σ:
- The volalility of the asset can be iteratively estimated using Vassalou and Xing (2002) procedure:
  - 1) Calculate $V_{t_0}(\sigma_V^n), ..., V_{t_N}(\sigma_V^n)$ from $S_{t_0}, ..., S_{t_N}$ by inverting Black Scholes.
  - 2) Estimate σ_V^{n+1} as:
    - $\sigma_V^{n+1} = \sqrt{\frac{1}{N\Delta t}\sum_{i=1}^{N}(\log(\frac{V_{t_i}}{V_{t_{i-1}}}) - \bar{\xi})^2}$ where $\bar{\xi} = \frac{1}{N\Delta t}\sum_{i=1}^{N}\log(\frac{V_{t_i}}{V_{t_{i-1}}})$

Example

`http:///35.180.63.47/Merton.xlsx`