# Interest Rate Risk

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## Introduction

- The interest rate risk is the risk that an investment's value will change due to a change in the absolute level of interest rates, in the spread between two rates, in the shape of the yield curve or in any other interest rate relationship.
- To hedge interest rate risk, we need to compute the price sensitivity of a portfolio of fixed-income securities with respect to the **term structure of interest rates** or the **yield curve**.

## Asset-Liability Management

- The main role of the asset-liability management (ALM) of a bank is to ensure that the **net interest margin** remains roughly constant through time.
  - The **net interest margin** is the ratio of the **net interest income** to the **income-producing assets**
  - The **net interest income** is the difference between the interest received (e.g. mortgage rate offered) and interest paid (e.g. interest on saving account).
- Banks need to adjust the interest received and interest paid to match assets and liabilities, known as **liquidity preference theory**.
  - It leads to long-term rates being higher than short-term rates.
  - They often use interest rate swaps to manage their exposures.
- In addition to eroding net interest margin, a mismatch of assets and liabilities can lead to **liquidity problems**:

- A bank that funds long-term loans with short-term deposits has to replace maturing deposits with new deposits on a regular basis (**rolling over** the deposits).

## Type of rates

- **Treasury rates**: issued by governments and assumed risk free.
- **LIBOR**: unsecured short-term borrowing rate between banks (one day to one year) used as a reference rate for interest rate swaps
- **Overnight indexed swap (OIS)** is a swap where a fixed interest rate for a period (1-month, 3-months, 1-year, 3-years) is exchanged for a the geometric average of overnight rates during the period.
  - USA: **Fed funds rate**; Europe: **Eonia**; UK: **SONIA**
  - used as the risk free rate for discounting cash flows.
  - key indicator of stress in the banking system: LIBOR-OIS spread (in "normal" condition: 10bps)
- **Repo rates**: secured borrowing rates from repurchase agreements. Usually a few basis points below LIBOR.

## Measures of Interest Rate Risk

- **Duration** and **Convexity** are measures of sensitivity of the price of a bond to interest rates.
- Duration and Convexity can also be used to measure the sensivitity of the price of a portfolio of fixed income securities.
- Duration of a bond:
  - measures the sensitivity of the price (value of principal) of a fixed-income investment to a change in interest rates.
- Convexity of a bond:
  - measures the degree of non-linear relationship between the price and the yield of a bond.

## Duration of a Bond

- Four types of durations are defined that differ in the way they account for interest rate changes, bond's embedded options and redemption features:
  - Macaulay duration
  - Modified duration
  - Effective duration: duration calculation for bonds that have embedded option
  - Key-rate duration: duration at a specific maturity point along the entirety of the yield curve

## Bond yield

Suppose that a bond provides cash flows $c_1$, $c_2$, $\ldots$ $c_{n-1}$ at time $t_1$, $t_2$, $\ldots$ $t_{n-1}$ and $c_n$ at maturity. Let us define $B$ the market price of a bond.

- The bond yield $y$ is defined as the discount rate that equates the bond's theoretical price to its market price, mainly:

$\sum_{t=1}^{n-1} \frac{C_t}{(1+y)^t} + \frac{C_n}{(1+y)^n} = B$

## Duration

- The duration $D$ of a bond is defined as: $D = -\frac{1}{B}\frac{dB}{dy} \approx -\frac{1}{B}\frac{\Delta B}{\Delta y}$
- If we define the present value of all cash flows $c_i$ including the principal repayment as $\nu_i$, market value of the bond is $D = \sum_{t=1}^{n} \nu_i$.
  - The duration of the bond is the **Macaulay's duration**: $D = \sum_{i=1}^{n} t_i \frac{\nu_i}{B}$
  - The duration is therefore the weighted average time payments are made.
  - When the bond is measured with continuous compounding, the **Macauley's duration** equals the **duration**.

## Modified Duration and Dollar Duration

- When the bond yield $y$ is measured with annual compounding, the **Macauley's duration** must be dividend by $1 + y$.

- Durations defined with these adjustments are referred to as **modified duration**.

- The **Dollar duration** of a bond is defined as the product of its duration and its price: $D_d = -\frac{dB}{dy}$.

## Convexity

- The duration is the sensitivity of the bond price with respect to the yield. As the yield curve is convex, the duration changes with changes of the yield.

- The convexity of a bond is defined as:

$$C = \frac{1}{B}\frac{d^2B}{dy^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B} \tag{1}$$

when $y$ is the bond yield measured with continuous compounding.

- The change in the bond price is $\Delta B = \frac{dB}{dy}\Delta y + \frac{1}{2}\frac{d^2B}{dy^2}\Delta y^2$ so $\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$

- The **dollar convexity** is $C_d = \frac{d^2B}{dy^2}$

## Portfolio duration and convexity

- The definitions of duration and convexity can be generalised to be applied to portfolios of bonds or interest-rate-dependent instruments.

- We define a **parallel shift** in the zero-coupon yield curve as a shift where all zero-coupon interest change by the same amount.

- Suppose that $P$ is the value of the portfolio of interest-rate-dependent securities. If we make a small parallel shift in the yield curve $\Delta y$, we can observe a change $\Delta P$ in P.

- Given a portfolio of $n$ assets $P = \sum_{i=1}^{n} X_i$, the portfolio **duration** is $-\frac{1}{P}\frac{\Delta P}{\Delta y} = -\frac{1}{P}\sum_{i=1}^{n}\frac{\Delta X_i}{\Delta y}$.
- By the same token, the relation price change of the portfolio becomes $\frac{\Delta P}{P} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$

## Portfolio immunization

- A portfolio consisting of long and short positions in interest-rate-dependent assets can be protected against relatively small parallel shifts in the yield curve by ensuring that its duration is zero.
- It can be protected against relatively large parallel shifts in the yield curve by ensuring that its duration and convexity are both zero or close to zero.

## Non-parallel yield curve shifts

- The relative price change of the portfolio as a function of the duration and convexity only holds for **parallel shifts** of the yield curve $y$.
- The **partial duration** measure can be computed by shifting only one point on the yield curve. The partial duration of the portfolio for the $i^{th}$ point on the zero curve is $D_i = -\frac{1}{P}\frac{\Delta P_i}{\Delta y_i}$
- The sum of all partial duration equals the duration measure.

## Bucket Deltas

- A variation on the partial duration approach consists of dividing the yield curve into a number of **segments** or **buckets** and computing the dollar impact of changing all zero rates corresponding to the bucket by 1bp while keeping all other zero rates unchanged.
- The method is called **GAP management**. The sum of all deltas for all segments is known as **DV01**.

## Principal Component Analysis (PCA)

- The approach presented requires the computation of 10 or 15 different deltas for each zero curve. For large portfolios of interest rate derivatives in several currencies, the number of computations are often prohibitive.

- One approach to handle risk arising from group of highly correlated market variables is **principal component analysis**:
  - It takes historical data on daily changes in the market variables and, by projecting them on their highest eigenvectors, define a set of components or factors that explain the movements.
  - Each interest rate change is then expressed as a linear sum of the factors by solving a system of $n$ equations, one per component.
  - The quantity of a particular factor in the interest rate changes on a particular day is known as the **factor score** for the day and its importance is measured by its standard deviation.
  - The risk the portfolio of interest-rate-dependent instruments is related to the movements of the principal components.